Optimization of Transmission Power in Competitive Wireless Networks

Patrick Maillé[1] and Bruno Tuffin[2]

[1] Institut Telecom; Telecom Bretagne
2 rue de la Châtaigneraie CS 17607, 35576 Cesson Sévigné Cedex, France
Université européenne de Bretagne
patrick.maille@telecom-bretagne.eu

[2] INRIA Rennes - Bretagne Atlantique, Campus universitaire de Beaulieu
35042 Rennes Cedex, France
btuffin@irisa.fr

Abstract. Competition among providers has become an important issue in current and future wireless telecommunication networks. The providers may operate using different technologies, such as WiFi, WiMAX, UMTS... In a previous work, we have analyzed the price competition among two providers, one operating in only a subdomain of the other, due to smaller distance range. A typical situation is WiFi against WiMAX. We propose here to add a supplementary level of decision on top of that game, making use of its equilibrium: the smaller-range provider plays with its transmission power in order to attract more customers and potentially increase its revenue. We determine the optimal power in the case where energy has a negligible cost, as well as when its cost is linear in transmission power.

Keywords: Economics, Competition, Wireless Networks, WiFi, WiMAX.

1 Introduction

Telecommunication networks, and especially wireless networks, have experienced an increase in terms of traffic and subscriptions, but at the same time a fierce competition among providers. Demand for service is distributed among competitors based on the access price and available Quality of Service (QoS). Pricing strategies therefore form an important parameter in the competition. Up to now, pricing has received a large interest in the networking community, due to resurce scarcity with respect to demand. Pricing is justified by its capacity to control demand (and therefore the QoS), and/or to differentiate services [1–3]. Most models investigate optimal pricing strategies in the case of a *monopoly*, whereas an *oligopoly*, with several providers fighting for customers, could drive to substantially different results, as highlighted in [4]. Thus competition requires a deeper attention. Notable first attempts in this direction can be found, not exhaustively, in [5–8].

This paper pertains to that stream of work. We consider two providers, denoted by provider 1 and provider 2, in competition for customers, where provider 2

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is assumed to operate in a subdomain of provider 1's access area. This is typically the case of a WiMAX operator (provider 1) against a WiFi one (provider 2). Each provider fixes a price, and demand is distributed according to the classical Wardrop principle [9] described later on.

In a previous paper [10], the subdomain covered by provider 2 was assumed to be fixed, and existence and uniqueness of a Nash equilibrium on prices played by providers were proved. Recall that a Nash equilibrium characterizes a profile of strategies such that no provider can improve its utility (revenue) without changing *unilaterally* its own strategy. It is therefore a point from which we do not deviate in the case of selfish providers. In that paper, using a particular pricing scheme, the price of anarchy was also proved to be one, meaning that there is no loss of social welfare due to user and provider selfishness with respect to an optimal cooperative case.

The present work is built on the results in [10]. Our goal is to investigate what happens if the smaller provider can initially choose its transmission power before the above game is played, and therefore if he can reach, and potentially attract, more customers from the larger provider in order to increase its revenue. We study this problem when there is no cost related to the transmission, and when such a cost exists. The basic question we want to solve is: is there an interest for the smaller user to be competiting over the whole domain? The answer is no in general, even if there is no cost associated to transmission power: actually in this case, there is no gain of revenue, even if no loss either, after reaching a given threshold.

The paper is organized as follows. Section 2 describes the model, its assumptions, and the main results presented in [10]. Section 3 then studies the optimal decision of provider 2 (the smaller one) in terms of the proportion of area he can reach on the domain of provider 1 (directly related to transmission power), in order to maximize its revenue. Section 4 does the same kind of analysis, but in the case where there is a cost associated to the power for transmitting data. Finally we conclude and give few research directions in Section 5.

2 Model and Previous Results

2.1 Model

We consider two providers, denoted by 1 and 2, with provider 2 operating in a subdomain of provider 1, as illustrated in Figure 1. We call zone B the coverage region of provider 2. Zone A stands for the region where only provider 1 operates. This is a typical situation of a WiFi provider operating on smaller distances -tens of meters- than a WiMAX one -covering many kilometers-.

Competition is analyzed on a simplified model, where time is discretized, divided into slots. Let C_i be the capacity of provider i ($i \in \{1, 2\}$), i.e., the number of packets he can serve during one slot. Let d_i be the demand experienced by provider i in a given slot. If $d_i \leq C_i$, all packets are served but as soon as $d_i > C_i$, only C_i are served and the $d_i - C_i$ rejected ones are chosen uniformly. Compacted, each packet is served with probability $\min(C_i/d_i, 1)$.

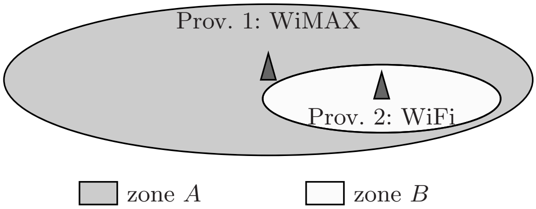

Fig. 1. The competition framework

Each provider fixes an *access* price (p_i for provider i) paid a soon as a packet is submitted, independently of being rejected or transmitted. This produces an incentive to limit the amount of sent packets. Indeed, the *perceived price* $\bar{p}_i$ per packet, i.e., the expected price to successfully send a packet, is

$$\bar{p}_i = p_i / \min(C_i/d_i, 1) = p_i \max(d_i/C_i, 1). \tag{1}$$

This kind of model was already studied in [11] for a single provider (a monopoly), when priority classes are defined and charged with different prices. The case of an oligopoly (with single service classes) is analyzed in [12]. The present model with one provider operating in a sub-area of the other is described in [10], where the smaller one does not play on its transmission range. Our basic model therefore follows [10], as described now.

Taking into account the point of view of the users, total demand on the whole domain depends on the perceived price $\bar{p}$, and is assumed to be a continuous and strictly decreasing function $D(\cdot)$ on its support $[0, p_{\max})$, with possibly $p_{\max} = +\infty$. Of course, we assume some potential congestion, i.e., $D(0) > C_1 + C_2$ to avoid uninteresting cases.

In order to introduce an additional and useful notation, remark that providing a demand function D is equivalent to providing a *marginal valuation function* $v : q \mapsto \inf\{p : D(p) \leq q\}$ (with the convention $\inf \emptyset = 0$), representing the maximum unit price at which q trafic units can be sold:

$$v(q) = \begin{cases} D^{-1}(q) & \text{if } q \in (0, D(0)) \\ p_{\max} & \text{if } q = 0 \\ 0 & \text{if } q \geq D(0). \end{cases} \tag{2}$$

Since D is nonincreasing, neither is v.

We finally define α as the proportion of the population covered by zone B. To simplify our analysis in next section, we will assume users uniformly distributed over the domain, and the domains being delimited by circles, with the restriction that the disc covered by provider 2 is always included in that of provider 1. Nonetheless, the general results of next subsections do not need such restrictions. In any case, the demand function in zone A is $(1-\alpha)D(\cdot)$, while it is $\alpha D(\cdot)$ in zone B, assuming equidistribution of users' willingness-to-pay accross subdomains.

User demand is assumed to split among providers following Wardrop's principle [9], which states all users choose the available provider with the least perceived price, and none if this perceived price is too high. Moreover, the demand on each zone is a function of the minimum perceived price available in that zone. Those conditions are a limit approximation of the Nash equilibrium conditions for the noncooperative game played among users, when the individual weight of each user tends to zero, i.e. no individual user can unilaterally impact the perceived price of the providers. This kind of game is called *nonatomic*, since it corresponds to each user being infinitely small.

Formally, let $d_{1,A}$, the demand experienced by provider 1 in zone A, and $d_{1,B}$, the demand in zone B, with $d_1 = d_{1,A} + d_{1,B}$. For given prices (p_1, p_2) set by the providers, the conditions imposed by Wardrop's principle lead to (a set of) perceived prices $(\bar{p}_1, \bar{p}_2)$, and demands $d_{1,A}, d_{1,B}, d_2$, called a Wardrop equilibrium, that satisfy

$$\bar{p}_1 = p_1 \max\left(1, \frac{d_{1,A} + d_{1,B}}{C_1}\right) \tag{3}$$

$$\bar{p}_2 = p_2 \max\left(1, \frac{d_2}{C_2}\right) \tag{4}$$

$$d_{1,A} = (1-\alpha)D(\bar{p}_1) \tag{5}$$

$$d_{1,B} + d_2 = \alpha D(\min(\bar{p}_1, \bar{p}_2)) \tag{6}$$

$$\bar{p}_1 > \bar{p}_2 \Rightarrow d_{1,B} = 0 \tag{7}$$

$$\bar{p}_1 < \bar{p}_2 \Rightarrow d_2 = 0. \tag{8}$$

Relations (7) and (8) come from zone B users choosing the cheapest provider, while (5) and (6) are the demand-price relations for each zone.

2.2 Previous Analysis

In [10], we have shown the following results:

- For each price profile (p_1, p_2), there exists at least one Wardrop equilibrium. Moreover, the corresponding perceived prices $(\bar{p}_1, \bar{p}_2)$ are unique. The only cases when demands might not be unique are when $\bar{p}_1 = p_1 = p_2 = \bar{p}_2$ and at the same time $d_1 + d_2 < C_1 + C_2$.
- We consider the non-cooperative game, where providers play with their price to maximize their own revenue $R_i(p_1, p_2) := p_i d_i$ knowing that demand will spread according to the above Wardrop equilibrium. Under the common assumption that price elasticity of demand $-\frac{D'(p)p}{D(p)}$ is strictly larger than 1 for all $p \in [\hat{p}, p_{\max})$, with $\hat{p} := \min\left(v\left(\frac{C_1}{1-\alpha}\right), v\left(\frac{C_2}{\alpha}\right)\right)$, there *exists* a *unique*

Nash equilibrium (p_1^*, p_2^*) in the price war between providers[1]. That Nash equilibrium is characterized as follows.

- If $\frac{C_1}{1-\alpha} < \frac{C_2}{\alpha}$ (i.e. $\alpha < \frac{C_2}{C_1+C_2}$), the Nash equilibrium is such that

$$p_1^* = v\left(\frac{C_1}{1-\alpha}\right) < p_2^* = v\left(\frac{C_2}{\alpha}\right). \quad (9)$$

  We then have $d_{1,A} = C_1$, $d_{1,B} = 0$ and $d_2 = C_2$, meaning that demand exactly equals capacity and zone B is left to provider 2 by provider 1.
- If $\frac{C_1}{1-\alpha} > \frac{C_2}{\alpha}$ (i.e. $\alpha \geq \frac{C_2}{C_1+C_2}$), the Nash equilibrium is such that prices are the same

$$p_1^* = p_2^* = p^* = v(C_1 + C_2). \quad (10)$$

  We then have $d_2 = C_2$, $d_{1,A} + d_{1,B} = C_1$. Again, demand exactly equals capacity, but zone B is shared by the providers (except for the limit case $\alpha = \frac{C_2}{C_1+C_2}$, when only provider 2 has customers in zone B).

– The Price of Anarchy, defined as the worst-case ratio comparing social welfare (sum of valuations of all actors) at the Nash equilibrium to the optimal value, is equal to one: social welfare is maximized even in the presence of selfish users and providers.

3 Optimal Radius/Proportion Parameter without Any Cost

We now assume that provider 2 can play with its transmission power, i.e. with the proportion parameter α representing the proportion of customers that it can actually reach (in bijection with transmission power), and for which he will be in competition with provider 1. The idea is to play strategically and *use* the information about what the Nash equilibrium is for each value of α. We therefore end up with a two-stage game where provider 2 plays first on α, and then both providers play on prices. Provider 2 acts as a leader of a kind of Stackelberg game [13], since we assume it uses the knowledge of the pricing game outcome (that we described earlier) to perform its best choice of α.

We further assume to simplify the analysis that the antennas of the two providers are located at the same point. Then, increasing the range of action of the smaller one will always let him in a subdomain of the big one (or the antennas do not need to be located at the same point provided the domain covered by 2 is included in that covered by 1).

In this section, we assume that provider 2 payoff is not affected by the power it uses. The other case is considered in next section. In this situation, from (9)

[1] It this paper we will play on α and still consider that demand elasticity is larger than 1 even if $\hat{p} = \min\left(v\left(\frac{C_1}{1-\alpha}\right), v\left(\frac{C_2}{\alpha}\right)\right)$ changes.

and (10) we have at Nash equilibrium $d_2 = C_2$, and provider 2 revenue is expressed as a function of α by

$$R_2(\alpha) = \begin{cases} v\left(\frac{C_2}{\alpha}\right) C_2 & \text{if } \frac{C_1}{1-\alpha} \leq \frac{C_2}{\alpha} \\ v(C_1 + C_2)C_2 & \text{if } \frac{C_1}{1-\alpha} > \frac{C_2}{\alpha}, \end{cases}$$

which is equivalent to

$$R_2(\alpha) = C_2 v \left(\max \left(\frac{C_2}{\alpha}, C_1 + C_2 \right) \right). \tag{11}$$

It is therefore constant as soon as $\alpha \geq \frac{C_2}{C_1+C_2}$. Note also that it is a continuous function of α.

On $[0, \frac{C_2}{C_1+C_2}]$, function R_2 is increasing due to the non-increasingness of v. The optimal choice is therefore any value $\alpha \in [\frac{C_2}{C_1+C_2}, 1]$, all producing the same revenue $v(C_1 + C_2)C_2$.

It is interesting to us to remark that there is no increase in revenue after the threshold $\alpha = C_2/(C_1+C_2)$ is reached. Indeed, in that case, demand and optimal price will be the same whatever α. On the other hand, from practical reasons, $\alpha = \frac{C_2}{C_1+C_2}$ is the most relevant choice because of more limited interferences and power consumption.

4 Optimal Radius/Proportion Parameter with Transmission Power Cost

We now assume that increasing the transmission power P induces a cost linear in that power, βP with β a constant. An important remark is that for the pricing game with fixed α, this has no consequence because it is a constant cost and therefore does not change the results.

If R is the radius such that provider 2 can transmit with a minimal reception power P_{min} for a given QoS, and assuming without loss of generailty that the coverage radius of provider 1 equals 1, we have $\alpha = \pi R^2/\pi = R^2$.

Similarly, we assume that for a user located at a distance d of the antenna, the reception power is $c\frac{P}{d^\mu}$ with c and μ constants (the value of μ depends on the area -countryside, city...-, but generally $2 \leq \mu \leq 5$). In order to fulfill a required minimal value P_{min} at reception, the relation between power and radius is

$$P_{min} = c\frac{P}{R^\mu}$$

i.e., $R^\mu = \frac{c}{P_{min}}P = \alpha^{\mu/2}$, which yields $P = \frac{P_{\min}}{c}\alpha^{\mu/2}$.

The goal is therefore to find the value α maximizing the overall benefit B_2, that is the revenue (11) at Nash equilibrium minus the power cost:

$$\begin{aligned} B_2(\alpha) &= R_2(\alpha) - \beta P \\ &= C_2 v \left(\frac{C_2}{\alpha} \right) - \frac{\beta P_{min}}{c} \alpha^{\mu/2}. \end{aligned}$$

We then have the following result.

Proposition 1. *If v is derivable and concave on its support, and $\mu \geq 2$, there is a unique solution $\alpha^* \in [0,1]$ for optimizing the net revenue $B_2(\alpha)$ of provider 2. Moreover, $\alpha^* \in [0, \frac{C_2}{C_1+C_2}]$*

Proof. Any optimal value of α is necessarily in $[0, \frac{C_2}{C_1+C_2}]$ because revenue is constant in $[\frac{C_2}{C_1+C_2}, 1]$ while cost strictly increases.

On $[0, \frac{C_2}{C_1+C_2}]$, the maximization problem writes

$$\max_{\alpha \in \left[0, \frac{C_2}{C_1+C_2}\right]} v\left(\frac{C_2}{\alpha}\right) C_2 - \frac{\beta P_{min}}{c} \alpha^{\mu/2}. \tag{12}$$

The derivative of the objective function is

$$-(1/\alpha^2) v'\left(\frac{C_2}{\alpha}\right)(C_2)^2 - \frac{\beta \mu P_{min}}{2c} \alpha^{\mu/2-1},$$

which is strictly decreasing in α on the support of $v(C_2/\cdot)$, thus the objective function is strictly concave on that support. Remark that possible values beyond the support of $v(C_2/\cdot)$ are not of interest since the associated objective is strictly negative.

Trying to find if there is a value of α for which that derivative is zero gives $-v'\left(\frac{C_2}{\alpha}\right) = \frac{\beta \mu P_{min}}{2cC_2^2} \alpha^{\mu/2+1}$. Due to the strict concavity of the objective, if an interior solution exists then it is unique. Otherwise, the derivative is always of the same sign, meaning that the optimal value is obtained at one of the extremities of the interval $[0, \frac{C_2}{C_1+C_2}]$.

Proposition 1 establishes that only one α will be chosen by provider 2. We now investigate the consequences of that choice on the price perceived by providers. More precisely, from (9) and (10) we know that at the equilibrium of the price war, the perceived prices $\bar{p}_1$ and $\bar{p}_2$ respectively equal the real prices p_1 and p_2, and that

- if $\alpha \geq \frac{C_2}{C_1+C_2}$ then $p_1 = p_2 = v(C_1 + C_2)$;
- if $\alpha < \frac{C_2}{C_1+C_2}$ then $p_1 = v\left(\frac{C_1}{1-\alpha}\right) > p_2 = v\left(\frac{C_2}{\alpha}\right)$.

In other words, when provider 2 really provides service (i.e. the optimal value α^* is strictly positive), then if the optimization problem (12) has an interior solution, the equilibrium perceived price in zone B, namely p_2, is strictly below the equilibrium perceived price p_1 in zone A. Otherwise, the perceived price on both zones is the same and equals $v(C_1 + C_2)$.

Those considerations are summarized in the next proposition.

Proposition 2. *Assume that provider 2 has an interest in providing service, i.e. that there exists $\alpha \in \left(0, \frac{C_2}{C_1+C_2}\right]$ such that*

$$v\left(\frac{C_2}{\alpha}\right) C_2 - \frac{\beta P_{min}}{c} \alpha^{\mu/2} > 0.$$

Then,

- *either* $-v'(C_1+C_2) \geq \frac{\beta\mu P_{\min}}{2c} C_2^{\mu/2-1}(C_1+C_2)^{-\mu/2-1}$, *therefore* $\alpha^* = \frac{C_2}{C_1+C_2}$ *and users in zones* A *and* B *perceive the same price at equilibrium;*
- *or* $-v'(C_1+C_2) < \frac{\beta\mu P_{\min}}{2c} C_2^{\mu/2-1}(C_1+C_2)^{-\mu/2-1}$, *therefore* $\alpha^* < \frac{C_2}{C_1+C_2}$, *and users in zone* B *will all choose provider* 2 *and experience a strictly lower price than zone* A *users.*

Proof. We just express the condition for the derivative of the objective function in (12) to be positive or negative at $\alpha = \frac{C_2}{C_1+C_2}$.

Example. Assume that $v(q) = 10 - q$ over [0,10], $C_1 = C_2 = 1$, $\mu = 2$ and $\frac{\beta P_{min}}{c} = 16$ for convenience.

The objective function $R_2(\alpha) - \frac{\beta P_{min}}{c}\alpha^{\mu/2}$ becomes on $[0, 1/2]$

$$10 - \frac{1}{\alpha} - 16\alpha.$$

The optimal value of $R_2(\cdot)$ is 2, obtained at $\alpha^* = 1/4 < \frac{C_2}{C_1+C_2}$. For that value of α, the price war among providers leads to the prices $p_1 = v(C_1/(1-\alpha)) = 26/3 \simeq 8.67$ and $p_2 = v(C_2/\alpha) = 6$. Therefore all users in zone B choose (the cheaper) provider 2.

5 Conclusion

In this paper, we have worked on determining the optimal proportion of customers of a concurrent that a smaller provider should try to reach in order to maximize its revenue. We have shown, based on results on the price war from a previous paper, that if there is no cost associated to power consumption, there is a threshold over which revenue stops increasing. It means that there is no need to install competition on the whole domain, i.e. for all customers. When increasing the transmission range induces a cost, assuming concavity of the marginal valuation function, we have proved the existence and uniqueness of the optimal range value. Determing it can be done very simply numerically.

We plan to work further on that kind of model. What happens if one of the providers is not included in the domain of the other? What if there is uncertainty on demand?

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