A Pricing Model for a Mobile Network Operator Sharing Limited Resource with a Mobile Virtual Network Operator

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Abstract. Radio spectrum allocation is essential to the provision of mobile communication services. The spectrum is a finite resource and can accomodate a limited number of simultaneous users at one time. Due to this scarcity, allocating traditional mobile licenses to new mobile operators is unrealizable. Hence, new entrants should bargain access to the networks of the incumbents who establish contracts specifying access charge and maximum traffic volume that the MVNO is allowed to send on the MNO's network. In this article a Mobile Network operator (MNO) shares his finite network resource with a Mobile Virtual Network operator (MVNO) lacking the infrastructure. We study the game where the MVNO invests in content/advertising to compensate for the quality of service degradation. Modeling the system as a supply-chain, i.e. a logistics network consisting of the MNO, the MVNO and the consumers, we determine the access charge and the optimal traffic volume that the MVNO should be allowed to send on the MNO's network to coordinate the system.

1 Introduction

Despite the absence of common definition, Mobile Virtual Network operators (MVNOs) are characterized as being mobile operators without their own infrastructure and government issued licenses. MVNOs buy spectrum and possibly also infrastructure from primary providers, referred to as Mobile Network operators (MNOs). They increase competition in the retail market and enable MNOs to fetch niche markets. Since they are new in a highly competitive market, MVNOs invest a lot in advertising campaigns or specific contents (for instance, M6 Mobile a MVNO on the Orange network, broadcasts soccer games; while Fnac Mobile proposes live concerts or advices about expositions). However, up to date, MVNOs are not really powerfull in the retail market. Indeed most contracts linked MNOs with MVNOs are too restrictive.

Deregulation provides wireless cellular network operators incentives to sell their exceeding spectrum for secondary usage. As an example, Mutlu et al. [5] introduce two classes of consumers: primary users (PUs) whose demand function is price independent since they have long-term contracts with the operator and secondary opportunistic users(SUs). The cellular operator (MNO) aims at selling his exceeding spectrum to

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increase his revenue but he should be careful to not decrease the quality of service perceived by the PUs who have long term contracts with him. Indeed SU's presence can increase PU's blocking probability, generating a punishement, i.e. the loss of market shares. However they do not introduce competition between the operators on the consumers.

Sethi et al. [4] outline the growing importance of the advertising in most industries today. To increase the advertising investment level, cooperative advertising is used, i.e. producers and sellers share the cost of advertising and the resulting revenue. Contrary to Sethi et al. [4] who use a dynamic demand process based on word-of-mouth effects and advertising perception, our demand function results from a fixed point equation. Indeed it relies on the *opportunity cost* [9] which incorporates parameters such as price, delay and content/advertising investment level. Besides, contrary to them, we test various forms of contracts and there is some competition between the manufacturer (MNO) and the retailer (MVNO) on consumers.

In this article, we deal with a MVNO interconnected with a MNO, transmitting data-services including Voice-over-IP. Each operator is supposed to have a fixed market share, i.e. there is no competition on the consumers, but on the proportion of resource that is shared (bandwidth). Following [4], the system is modeled as a supply-chain. According to [7], *supply-chain management* is a set of approaches utilized to efficiently integrate suppliers (MVNOs), manufacturers (MNOs), and consumers, so that merchandise (i.e. network access) is delivered and shared at the right quantities, to the right consumers, and at the right time, in order to minimize systemwide costs while satisfying service level requirements (i.e. providing the operators sufficient profits). Relationships between the MNO and the MVNO can take many forms, both formal and informal, but often, to ensure adequate bandwith sharing and timely deliveries, MNOs and MVNOs typically agree on supply contracts. Besides these contracts can be used as powerful tools to achieve global optimization, and to motivate the supply-chain parties to reveal their true forecast of customer demand. In our case, the MNO aims at defining a contract allowing the MVNO to send some traffic on his limited resource network. The contract definition is fundamental. Indeed, if the MNO sells too much bandwidth to the MVNO, he might lack capacity for his own clients which would imply a quality of service degradation and in turn, a heavy loss of money. But if he does not sell enough bandwidth, some capacity might be unused which would generate also a heavy loss of money. Furthermore each operator wants to maximize one's revenue by determining consumers' retail prices.

We study the game when the MVNO acts as a Stackelberg leader by investing in advertsing to compensate for the quality of service deterioration, in Section 2. Then, in Section 3, we determine the maximum traffic volume that the MVNO should be allowed to send on the MNO's network and the associated access charge to coordinate the supply-chain. It means that the MNO should have no incentives to sell more or less traffic access to the MVNO and, that the MVNO should agree on the access charge that the MNO suggests.

For the sake of simplicity in all the game models, we will consider the MNO as a male player while the MVNO is female. Also, due to space constraints, most proofs are left to the extended version [11] of the paper. We just keep the proof of Lemma 1 as it is useful to get the main principle.

2 Optimization of Operators' Revenues without Competition on Consumers When the MVNO Invests in Advertising

In this section, the operators have a fixed number of consumers. They do not compete for consumer market shares but for the proportion of resource they are going to use. To emphasize that, assuming no competition for consumers is relevant in the cases where providers have targeted specific market segments; we can for instance think about a MVNO broadcasting soccer games on mobile phones, or other specific contents: it aims at providing a compensation for the MNO's QoS degradation. The operators want to increase the usage of each consumer they have acquired because it increases their average revenue per user (ARPU). However, the total traffic they can transport is limited because they share a common resource (i.e. spectrum because MNO's consumers and MVNO's consumers are in the same coverage area). The total available bandwidth is μ. We assume in addition that the MNO does not differentiate between his consumers' traffic and the MVNO's consumers' traffic. The MNO now has to share his network with the MVNO; indeed the MVNO is allowed to send a maximum traffic volume $\Lambda_2 \in [0; \mu]$, on the MNO's network. Her access charge is $w\Lambda_2$, where w is a wholesale access price. Besides the MNO saves $\Lambda_1 = \mu - \Lambda_2$ as capacity for his own traffic. The interconnexion model is decribed in Figure 1.

Both operators' consumers valuate the opportunity cost of consuming a unit of traffic (time wasted and price) and send traffic accordingly. We let $p_1,\ p_2 \geq 0$ be the prices paid by consumers for consuming a unit of traffic chosen by both operators. The opportunity costs perceived by consumers for one unit of traffic are $c_1(p_1, p_2) = p_1 + \alpha_1\ d(p_1, p_2)$ at the MNO and $c_2(p_1, p_2) = p_2 + \alpha_2\ d(p_1, p_2) + \beta\ \Theta(\lambda_2, \theta)$ at the MVNO, where $\alpha_1 \geq 0$ (resp. $\alpha_2 \geq 0$) measures the MNO's (resp. MVNO's) consumers' sensitivity to the QoS $d(p_1, p_2)$, and $\beta \leq 0$ describes the sensitivity to content / advertising. $\Theta(\lambda_2, \theta) = \theta\lambda_2 + r$ $(\theta \leq 0,\ r > 0)$ models the MVNO's content/advertising investment level perceived by the consumer as a function of her traffic λ_2. Indeed, we assume that the MVNO's investment level is linearly decreasing in the traffic flow generated from her clients. If the consumers produce lots of traffic, it means that they think that the MVNO's perceived quality is satisfactory, hence the MVNO do not need to invest too much in content/advertising to compensate for the bad QoS. On the contrary, if the consumers do not produce much traffic, it means that the experienced QoS is too bad; consequentely the MVNO should invest in content/advertising to seduce them with an attractive brand image and make the traffic volume increase. Besides the more the MVNO invests, the better her brand image will be. Hence for the MVNO's consumers, $-\beta\Theta\Big(\lambda_2, \theta\Big)$ which represents the brand image associated to the MVNO, compensates for the bad opportunity cost of traffic excessive delay.

Demand (that is, arrival rate or average traffic λ_k, for $k \in \{1, 2\}$) is driven by the (random) utility of a unit of traffic U_k, $\lambda_k = \Lambda_k \mathbf{P}\Big[U_k \geq c_k\Big]$ with c_k the above opportunity cost. More specifically, $\lambda_1 = \Lambda_1 \bar{F}\Big(p_1 + \alpha_1 d(p_1, p_2)\Big)$ for the MNO and

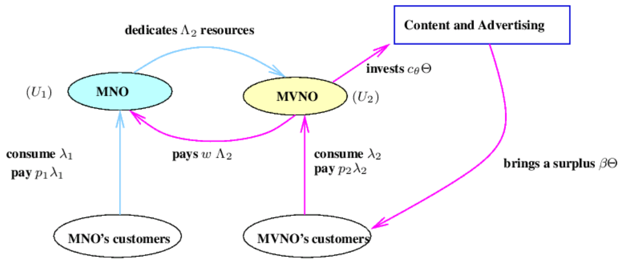

Fig. 1. Description of the interactions between the MNO and the MVNO when their market shares are fixed

$\lambda_2 = \Lambda_2\bar{F}\Big(p_2 + \alpha_2 d(p_1,p_2) + \beta\Theta(\lambda_2,\theta)\Big)$ for the MVNO, where $\bar{F}$ is the consumers' complementary cumulative distribution function modeling their opportunity cost perception. Indeed consumers send traffic only if their opportunity cost is inferior or equal to the utility they associate to the operator's QoS. We assume that the utility U is generated according to a uniform law on $[0;1]$. Hence, if there exists a solution with $\lambda_1, \lambda_2 > 0$ to the above fixed point equations, i.e. when the opportunity costs belong to $[0;1]$, it takes necessarily the form,

$$\lambda_1 = \Lambda_1\Big(1 - p_1 - \alpha_1 d(p_1,p_2)\Big) \text{ for the MNO} \tag{1}$$

$$\lambda_2 = \Lambda_2\Big(1 - p_2 - \alpha_2 d(p_1,p_2) - \beta\Theta(\lambda_2,\theta)\Big) \text{ for the MVNO.} \tag{2}$$

The QoS $d(p_1,p_2)$ is measured via the system's average delay. We assume that the network bottleneck is represented by an M/M/1 queueing system so that $d(p_1,p_2) = \frac{1}{\mu-(\lambda_1+\lambda_2)}$ where μ is the MNO's maximum bandwith volume [10]. Note that considering the steady-state delay is not a stringent constraint even if there is a feedback loop since prices depend on delay too. Indeed, if price changes operate at a much larger time scale, the queue reaches steady-state before the next price change, and the expected delay can be observed. This is assumed in the present paper.

The operators' utilities depend on their consumers' traffic and chosen prices. The MNO's utility U_1, is the sum of the revenue generated from his consumers and from the contract established with the MVNO minus the cost of his infrastructure C:

$$U_1(p_1,p_2) = p_1\lambda_1 - C + w\Lambda_2 \tag{3}$$

The MVNO's utility U_2, is the sum of the revenue generated from her consumers minus the contract and the content/advertising investment costs. We assume that the content/advertising investment costs are proportional to the investment level as perceived by each individual consumer. The cost of one unit of perceived content/advertising investment level (e.g. soccer game) is c_θ. As a result :

$$U_2(p_1,p_2) = p_2\lambda_2 - w\Lambda_2 - c_\theta\Theta(\lambda_2,\theta) \tag{4}$$

Lemma 1. *The fixed point Equations (1) and (2) have unique solutions in λ_1 and λ_2. Besides, the delay can be expressed as a function of both operators' prices p_1 and p_2:*

$$d(p_1,p_2) = \frac{\Lambda_1(1-p_1) + \frac{\Lambda_2}{1+\Lambda_2\beta\theta}\Big(1-p_2-\beta r\Big) - \mu + \sqrt{\Delta}}{2\Big[\alpha_1\Lambda_1 + \frac{\alpha_2}{1+\Lambda_2\beta\theta}\Lambda_2\Big]}, \tag{5}$$

where $\Delta = \Big[\mu - \Lambda_1(1-p_1) - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r)\Big]^2 + 4(\alpha_1\Lambda_1 + \frac{\alpha_2}{1+\Lambda_2\beta\theta}\Lambda_2)$.

Proof of Lemma 1. $\lambda_1 \mapsto \frac{\lambda_1}{\Lambda_1}$ is strictly increasing in λ_1, while $\lambda_1 \mapsto 1 - p_1 - \alpha\frac{1}{\mu-(\lambda_1+\lambda_2)}$ is decreasing in λ_1; then using the MNO's fixed point Equation (1) we infer that the traffic rate λ_1 is uniquely defined provided $p_1 + \alpha_1 d(p_1,p_2) \leq 1$ otherwise $\lambda_1 = 0$. Since $\lambda_2 \mapsto \frac{(1+\Lambda_2\beta\theta)\lambda_2}{\Lambda_2}$ is strictly increasing in λ_2 and $\lambda_2 \mapsto \bar{F}\Big(p_2 + \beta r + \alpha_2\frac{1}{\mu-(\lambda_1+\lambda_2)}\Big)$ is decreasing in λ_2; the Equation (2) tells us that the MVNO's traffic rate λ_2 is uniquely defined provided $p_2 + \alpha_2 d(p_1,p_2) \leq 1$ otherwise $\lambda_2 = 0$. The system delay can be written: $d(p_1,p_2) = \frac{1}{\mu-(\lambda_1+\lambda_2)} = \Big\{\mu - \Lambda_1(1-p_1) - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r) + (\alpha_1\Lambda_1 + \frac{\alpha_2\Lambda_2}{1+\Lambda_2\beta\theta})d(p_1,p_2)\Big\}^{-1}$. Determining analytically $d(p_1,p_2)$ is equivalent to solve a second order polynomial equation: $(\alpha_1\Lambda_1 + \frac{\alpha_2}{1+\Lambda_2\beta\theta}\Lambda_2)d(p_1,p_2)^2 + (\mu - \Lambda_1(1-p_1) - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r))d(p_1,p_2) - 1 = 0$. Since the discriminant $\Delta = \Big[\mu - \Lambda_1(1-p_1) - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r)\Big]^2 + 4(\alpha_1\Lambda_1 + \frac{\alpha_2}{1+\Lambda_2\beta\theta}\Lambda_2)$ is always non negative, we get that $d(p_1,p_2)$ is the unique positive root of the equation, which gives us Equation (5). □

In this section we assume that the contract parameters (w, Λ_2) are fixed, i.e. we deal with a simple form of wholesale contract; however it will be the purpose of Section 3 to study more complex contracts and define which one the MNO should choose depending on both operators' power relations and to optimize the contract parameters. Hence we assume that the MNO allows the MVNO to send Λ_2 as traffic volume on her network; while the MVNO pays a wholesale access price of $w\Lambda_2$ to gain access to the MNO's network. The operators' utility optimization problem is made of the two steps described below. It can be seen as a Stackelberg game where the MVNO is the leader since she has the possibility to invest in content/advertising and hence is more powerful. Indeed by investing in content/advertising, she will improve her brand image; as a result, the consumers' perceived opportunity costs will decrease, i.e. their traffic λ_2 will increase. In turn the MVNO's traffic increase implies that the MNO's network becomes more congested. Hence the MNO should have to lower his price to increase his consumers' traffic λ_1.

Operators' revenue optimization and game on content/advertising investment

1. The MVNO chooses a price p_2 and invests in content/advertising to maximize her utility, U_2;
2. the MNO chooses a price p_1 to maximize his utility, U_1;
3. the consumers compute their opportunity costs: $c_1(p_1,p_2)$ and $c_2(p_1,p_2)$, their traffics evolve according to the fixed point equations (1) and (2).

The game is solved by *backward induction*. The leader (MVNO) considers what the best response of the follower (MNO) is, i.e. how it will respond once it has observed the price p_2 and content/advertising investment level $\Theta(\lambda_2,\theta)$ of the leader. The leader then determines a price p_2 that maximizes her payoff, anticipating the predicted response of the follower. The follower actually observes this and in equilibrium picks the expected price p_1 as a response.

Proposition 1. *The MVNO's price being fixed to p_2*[1]*, to optimize the MNO's utility, we have to solve a fourth order polynomial equation*[2] *in p_1*

$$
\begin{aligned}
& a_4 p_1^4 + a_3 p_1^3 + a_2 p_1^2 + a_1 p_1 + a_0 = 0 \\
\Leftrightarrow & \Big[-3\alpha_1 \Lambda_1^6 \Big] p_1^4 + \Big[2KL\Lambda_1^2 - 2\alpha_1^2 \Lambda_1^5 M - 2\alpha_1^2 \Lambda_1^5 (\mu - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r) - \Lambda_1) \\
- & 2\alpha_1^2 \Lambda_1^4 P \Big] p_1^3 + \Big[K^2 \Lambda_1^2 + 2KLP + L^2 - \Lambda_1^4 \alpha_1^2 - 2\alpha_1^2 \Lambda_1^3 MP \\
- & \alpha_1^2 \Lambda_1^4 (\mu - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r) - \Lambda_1)^2 - 2\alpha_1 \Lambda_1^4 N \Big] p_1^2 + \Big[K^2 P + 2KLN \\
- & (\Lambda_1 + \alpha_1)^2 P - 2\alpha_1^2 \Lambda_1^3 MN - 2\Lambda_1 (\mu - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r) \\
- & \Lambda_1) \Big] p_1 + \Big[K^2 N - \Lambda_1 \alpha_1^2 N \Big] = 0,
\end{aligned}
$$

where we have set:

$$
\begin{aligned}
K &= 2[\alpha_1\Lambda_1 + \frac{\alpha_2}{1+\Lambda_2\beta\theta}\Lambda_2]\Lambda_1 - \alpha_1[\frac{\Lambda_1\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r)], \\
L &= -4\Lambda_1(\alpha_1\Lambda_1 + \frac{\alpha_2}{1+\Lambda_2\beta\theta}\Lambda_2) + 2\alpha_1\Lambda_1^2, \\
M &= \mu - \Lambda_1 - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r), \\
N &= [\mu - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r) - \Lambda_1]^2 + 4(\alpha_1\Lambda_1 + \frac{\alpha_2}{1+\Lambda_2\beta\theta}\Lambda_2), \\
P &= 2\Lambda_1(\mu - \frac{\Lambda_2}{1+\Lambda_2\beta\theta}(1-p_2-\beta r) - \Lambda_1).
\end{aligned}
$$

- *Besides if* $0 < w\Lambda_2 - C \leq \Lambda_1(1-p_1^*)\frac{d(1,p_2^*)}{d(p_1^*,p_2^*)}$, *a unique positive price maximizes the MNO's utility (3).*
- *If* $\alpha_1 > \frac{w\Lambda_2 - C}{\Lambda_1 d(1,p_2^*)}$ *then* $p_1^* \in [0;1]$. *Since the consumers' utilities belong to the interval* $[0;1]$*, to guarantee that the MNO's traffic rate* λ_1 *defined in Equation (1) does not vanish in* (p_1^*, p_2^*)*, i.e. that* $c_1(p_1^*, p_2^*) \in [0;1]$*, we impose that* $\alpha_1 \in]\frac{w\Lambda_2 - C}{\Lambda_1 d(1,p_2^*)}; \frac{1-p_1^*}{d(p_1^*,p_2^*)}]$.
- *If* $p_1^* \geq \frac{\Lambda_2}{\log \Lambda_2} - c_\theta\theta$ *then it is sufficient to choose* $\alpha_2 > \frac{p_2^* - \beta_2\Theta(\lambda_2(p_1^*,p_2^*),\theta)}{d(p_1^*,p_2^*)}$ *to guarantee that* $c_2(p_1^*, p_2^*) \in [0;1]$.

Proof of Proposition 1. The proof can be found in [11].
From this value, the optimal price for the MVNO can be derived numerically.

Numerical illustrations. Using Proposition 1, the MVNO's utility U_2 can be expressed as a function of p_2 since the MNO's consumer access price p_1 is now merely a function of p_2. We check numerically[3] in Figure 2 that U_2 has a unique solution in p_2. The

[1] It implies that the MVNO's content/advertising level is fixed according to the fixed point Equation (2).
[2] The polynomial equation coefficients ranked in decreasing power are $a_4,\ a_3,\ a_2,\ a_1,\ a_0$.
[3] In the revenue optimization problem, the parameters are defined as follows: $\alpha_1 = 0.8,\ \alpha_2 = 0.85,\ \mu = 10,\ \Lambda_1 = \Lambda_2 = 5,\ \beta = -0.4,\ \theta = -3,\ r = 16,\ C = 3,\ c_\theta = 0.5$.

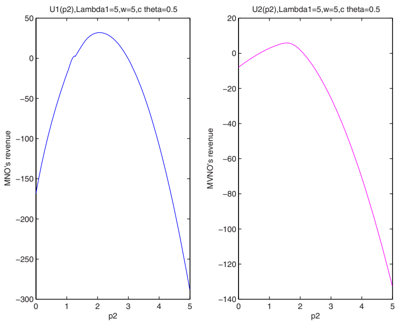

Fig. 2. The MVNO's price maximizing her utility is unique

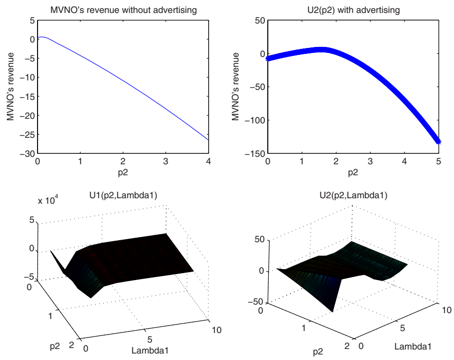

Fig. 3. Investing in content / advertising increases the MVNO's revenue and both operators' optimal revenues depend on the maximal traffic volume the MVNO is allowed to send on the MNO's network ($\Lambda_2 = \mu - \Lambda_1$)

maxima of the operators revenues are: $U_1^* = 22.01$ and $U_2^* = 5.89$. In Figure 3 we observe that both operators' maximized utilities relie on the maximum traffic volume the MVNO is allowed to send on the MNO's network[4] and that the MVNO has incentives to invest in content/advertising since maximized revenue is greater with some content/advertising than without ($U_2^*(\theta = 0, r = 0) = 1.19$).

3 Providers Power Relations and Contract Definition

The aim of this section is to define the contract between the MNO and the MVNO. Indeed we can see the MNO as a supplier of the scarce resource for both his consumers' business units and for the MVNO. Then the system composed of the MNO, the MVNO and both providers' consumers can be modeled as a supply-chain. The contracts of the supply-chain are between the MNO and the MVNO; additionally these latter determine optimal pricing strategies to sell their services on the market. The need from *coordination*, i.e. the maximization of the social-welfare using contract parameters imposed by a regulatory authority such that no provider has incentives to deviate, results from the following observation: if the MNO imposes upon the MVNO to buy $\Lambda_2^* = \arg\max_{\Lambda_2} U_1$, then the MVNO assumes all of the risk of having more bandwidth than she really needs, while the MNO takes no risk. Indeed, since the MNO takes no risk, he would like the MVNO to buy as much of his unused bandwith as possible; while the MVNO would prefer to limit the quantity due to the huge financial risk. Of course if the MVNO limits her order quantity there is a significant increase in the likelihood of spoilt capacity for the MNO. If the MNO is willing and able to share some of the risk with the MVNO, it may be profitable for the MVNO to order more bandwith, thereby reducing spoilt capacity probability and increasing profit for both of them. It turns out that a variety of

[4] It is quite natural since U_1 and U_2 are functions of Λ_1 and Λ_2.

supply-chain contracts enable this risk sharing, and therefore increase profits for both supply-chain entities [7]. In Section 2 we have supposed that the unit access charge w was fixed; i.e. we were in the simple case of a wholesale contract. The aim of this section is now to study more complex contracts such as revenue sharing, quantity discount and sales rebate [2]. The MNO wants to determine how much traffic the MVNO should be allowed to send on his network. He assumes to be in the *worst* case, i.e. that the MVNO has so much consumers that she sends the maximal allowed traffic volume on his network. More formally, we aim to determine the maximal traffic volume Λ_2 that the MVNO should buy at the access charge w such that neither of the operators can increase ones' profit by unilaterally deviating from his (her) choice.

If the MNO acts *selfishly*, he determines contract parameters: $\Lambda_2^*, w^* \equiv w(\Lambda_2^*)$ maximizing his own utility. However such a behavior is not optimal when looking at the social-welfare as the sum of MNO and MVNO utilities. Optimal performances can be achieved if they coordinate by contracting on a set of transfer payments such that each operator's objective becomes aligned with the supply-chain's objective. The contract coordinating the supply-chain relies on the optimal traffic volume $\Lambda_2^0 = \arg\max_{\Lambda_2} U$. In fact supply-chain's contracts play the role of an unbiased decision maker who would identify the best strategy for the entire supply-chain and allocate the whole profit between both players. This unbiased decision maker would consider the two supply-chain's partners: the MNO and the MVNO, as two members of the same organization. Hence the transfer of money between the parties would be ignored and the unbiased decision maker will maximize the supply-chain's profit. Contracts help firms to achieve global optimization [7]. The contract definition requires the four following steps:

(i) the MNO chooses a contract category between revenue sharing, quantity discount and sales rebate;
(ii) the MNO determines contract parameters w^*, Λ_2^* maximizing his utility U_1;
(iii) an unbiased decision maker computes $\Lambda_2^0 = \arg\max_{\Lambda_2} U$ and allocates the supply-chain's profit between the operators;
(iv) the MVNO is free to refuse the contract.

In practice, the MNO's share of the supply-chain's optimal profit is defined as in [2]: $\rho_1^* = \frac{U_1(\Lambda_2^*, w^*)}{U(\Lambda_2^0)}$, where the supply-chain's utility $U = U_1 + U_2$ is the sum of the MNO and MVNO's utilities. The higher this coefficient is, the more attractive the contract is for the MNO since $U_1 = \rho_1^* U(\Lambda_2^0)$ and $U_2 = (1 - \rho_1^*)U(\Lambda_2^0)$. We consider the three following types of contracts.

– If the revenue sharing contract is implemented, the MNO charges $w_\Phi(\Lambda_2)$ per unit traffic purchased plus the MVNO gives the MNO a percentage of her revenue (cf [3] for an application to the video cassette rental industry). Indeed in the sequential supply-chain, one important reason for the MVNO to refuse a contract is the high wholesale price [7]. If somehow the MVNO can convince the MNO to reduce the wholesale price, then clearly the MVNO will have an incentive to cooperate. Of course a reduction in wholesale price will decrease the MNO's profit if he is unable to sell more capacity. This is adressed in the revenue sharing contract where the MVNO and the MNO share the revenues generated from the consumers. Let $\Phi \in [0; 1]$ be the fraction of the supply-chain's revenue that the MVNO keeps;

$(1 - \Phi)$ is then the fraction earned by the MNO. Depending on both operators' power relations, the sharing parameter will be defined in Theorem 1. In the revenue sharing contract, the providers' utilities are defined as:

$$U_1 = (1 - \Phi)(\lambda_1 p_1 + \Lambda_2 p_2) + w_\Phi(\Lambda_2)\Lambda_2 - C \quad (6)$$
$$U_2 = \Phi(\lambda_1 p_1 + \Lambda_2 p_2) - w_\Phi(\Lambda_2)\Lambda_2 - c_\theta(\theta\Lambda_2 + r) \quad (7)$$

– In the quantity discount contract, the transfert payment is $w_d(p_2, \Lambda_2)\Lambda_2$ where w_d is a decreasing function of the MVNO's traffic Λ_2. The MVNO pays the MNO $w_d(p_2, \Lambda_2)\Lambda_2$ to send her traffic on the MNO's network [2]. In the quantity discount contract, the operators' utilities take the form:

$$U_1 = \lambda_1 p_1 + w_d(p_2, \Lambda_2)\Lambda_2 - C \quad (8)$$
$$U_2 = \Lambda_2 p_2 - w_d(p_2, \Lambda_2)\Lambda_2 - c_\theta(\theta\Lambda_2 + r) \quad (9)$$

– With a sales rebate contract, the MNO charges $w_s(p_1, p_2, \Lambda_2)$ per unit of traffic purchased by the MVNO but then gives the MVNO a rebate $r_s(\Lambda_2)$ which is a function of the traffic sent by the MVNO, if the MVNO's QoS $d(p_1, p_2)$ is above some predefined threshold d_0 [2]. In the sales rebate contract, the utilities are:

$$U_1 = \lambda_1 p_1 + w_s(p_1, p_2, \Lambda_2)\Lambda_2 - r_s(\Lambda_2)\mathbf{1}_{\{d(p_1,p_2)>d_0\}} - C \quad (10)$$
$$U_2 = \Lambda_2 p_2 - w_s(p_1, p_2, \Lambda_2)\Lambda_2 + r_s(\Lambda_2)\mathbf{1}_{\{d(p_1,p_2)>d_0\}} - c_\theta(\theta\Lambda_2 + r) \quad (11)$$

We insert the QoS measure expression, i.e. the delay $d(p_1, p_2) = \frac{1}{\mu - \left(\lambda_1 + \Lambda_2\right)}$, in the fixed point Equation (1); the MVNO's traffic being fixed to Λ_2. To determine the MNO's traffic $\lambda_1(\Lambda_2)$ we need to solve a second order polynomial equation in λ_1 whose unique positive root is:

$$\lambda_1(\Lambda_2) = \frac{1}{2}\Big\{[\mu - \Lambda_2 + \Lambda_1(1 - p_1)] + \Big((\mu - \Lambda_2 + \Lambda_1(1 - p_1))^2 - 4(\Lambda_1(1 - p_1) (\mu - \Lambda_2) - \alpha\Lambda_1)\Big)^{\frac{1}{2}}\Big\}. \quad (12)$$

Let $b \geq 1$ and $\gamma \in \mathbb{R}$ be coefficients characterizing the operators' power relations. We assume that the supply-chain's utlility is linear in the MNO's one, i.e. $U = bU_1 + \gamma$. It implies that the contract parameters which maximize the supply-chain's utility is a Nash equilibrium for the operators [2]. Indeed let (w, Λ_2) be contract parameters maximizing the supply-chain's utility U. If we assume that there exists Λ_2' such that $\Lambda_2' \neq \Lambda_2$ and $U_1(w, \Lambda_2') > U_1(w, \Lambda_2)$ then by linearity we get $bU_1(w, \Lambda_2') + \gamma > bU_1(w, \Lambda_2) + \gamma$ i.e. $U(w, \Lambda_2) > U(w, \Lambda_2')$. By application of Nash's Lemma [8] we get that the MNO's strategy is to choose Λ_2 with probability one. If we assume that there exists w' such that $w' \neq w$ and $U_2(w', \Lambda_2) > U_2(w, \Lambda_2)$ then by linearity we get $\frac{b}{b-1}U_2(w', \Lambda_2) - \frac{\gamma}{b-1} > \frac{b}{b-1}U_2(w, \Lambda_2) - \frac{\gamma}{b-1}$ i.e. $U(w', \Lambda_2) > U(w, \Lambda_2)$. By application of Nash's Lemma [8] we get that the MVNO's strategy is to choose w with probability one. Besides depending on both operators' power, we can express the MNO's utility has a linear function of the supply-chain's one: $U_1 = \frac{1}{b}U - \frac{\gamma}{b}$ and $U_2 = \frac{b-1}{b}U + \frac{\gamma}{b}$ for the MVNO. For each contract we determine $\Lambda_2^0(RS),\ \Lambda_2^0(QD),\ \Lambda_2^0(SR)$

i.e. the maximal traffic volume that the MVNO should be allowed to send on the MNO's network to maximize the supply-chain's utility under revenue sharing, quantity discount and sales rebate contracts. Then by substitution in the contract parameters we get $w^0_\Phi \equiv w_\Phi\Big(\Lambda^0_2(RS)\Big)$, $w^0_d \equiv w_d\Big(p_2, \Lambda^0_2(QD)\Big)$ and $w^0_s \equiv w_s\Big(p_1, p_2, \Lambda^0_2(SR)\Big)$.

Theorem 1. *The three contract parameters* $\Big(\Lambda^0_2(RS), w^0_\Phi\Big)$, $\Big(\Lambda^0_2(QD), w^0_d\Big)$ *and* $\Big(\Lambda^0_2(SR), w^0_s\Big)$ *which are respectively the MNO's optimal traffic to sell to the MVNO and the associated access charge for the revenue sharing, the quantity discount and the sales rebate contracts, defined with the following parameters*

$$\Phi = 1 - \frac{1}{b}, \text{ and } w_\Phi(\Lambda_2) = (\Phi - 1)\frac{\gamma + C\frac{\Phi}{\Phi-1} + c_\theta(\theta\Lambda_2 + r)}{\Lambda_2}; \tag{13}$$

$$w_d(p_2, \Lambda_2) = p_2 + \frac{c_\theta(\theta\Lambda_2 + r) + \gamma}{\Lambda_2}; \tag{14}$$

$$w_s(p_1, p_2, \Lambda_2) = \frac{(1-b)\lambda_1 p_1 + \Lambda_2 p_2}{b\Lambda_2}, \text{ and } r_s(\Lambda_2) = \frac{c_\theta(\theta\Lambda_2 + r) + \gamma}{b}; \tag{15}$$

coordinate the supply-chain (in the sense that it maximizes the total surplus of the MNO and the MVNO, i.e. the sum of their utilities).

Proof of Theorem 1. Using Equations (6) and (7), (8) and (9), (10) and (11); by identification of the relation $U = U_1 + U_2 = bU_1 + \gamma$, we infer the contract parameters values. □

We determine the optimal traffic volume that the MVNO should be allowed to send on the MNO's network to maximize the supply-chain's utility under the three contracts considered.

Proposition 2. *For the revenue sharing contract we get:*

$$\Lambda^0_2(RS) = \frac{1}{2 - \frac{1}{2[2\frac{(c_\theta\theta - \Phi p_2)^2}{\Phi p_1} + 1]^2}}\Bigg[2\mu - 2\Lambda_1(1-p_1) + \frac{1}{2[2\frac{(c_\theta\theta - \Phi p_2)^2}{\Phi p_1} + 1]^2}\Big(\mu - 3\Lambda_1(1-p_1)\Big) + \sqrt{\Delta_r}\Bigg], \tag{16}$$

where $\Delta_r = \Bigg[2\mu - 2\Lambda_1(1-p_1) + \frac{1}{2[2\frac{(c_\theta\theta - \Phi p_2)^2}{\Phi p_1} + 1]^2}\Big(\mu - 3\Lambda_1(1-p_1)\Big)\Bigg]^2 - 4\Bigg[\Big(2\frac{(c_\theta\theta - \Phi p_2)^2}{\Phi p_1} + 1\Big)^2\Bigg]\Bigg[\mu^2 - 2\mu\Lambda_1(1-p_1) + \Lambda_1^2(1-p_1)^2 + 4\alpha\Lambda_1 - \frac{1}{4[2\frac{(c_\theta\theta - \Phi p_2)^2}{\Phi p_1} + 1]}\Bigg]$.

For the quantity discount and sales rebate contracts we have:

$$\Lambda^0_2(QD) = \Lambda^0_2(SR) = \frac{1}{2}\Bigg[\frac{16}{(\frac{p_1}{2} - p_2 + c_\theta\theta)^2} - \Big(2\mu_2 - 2\Lambda_1(1-p_1) + 4\Lambda_1(1-p_1)\Big) + \sqrt{\Delta_q}\Bigg] \tag{17}$$

where $\Delta_q = \left[\left(2\mu_2 - 2\Lambda_1(1-p_1) + 4\Lambda_1(1-p_1)\right)\frac{16}{(\frac{p_1}{2}-p_2+c_\theta\theta)^2}\right]^2 - 4\left[\mu^2 - 2\Lambda_1(1-p_1)\mu + \Lambda_1^2(1-p_1)^2 + 4\alpha\Lambda_1 + \frac{16}{(\frac{p_1}{2}-p_2+c_\theta\theta)^2}\right]$.

To determine $\Lambda_2^*(RS)$ *(resp.* $\Lambda_2^*(QD)$*,* $\Lambda_2^*(SR)$*) we substitute* $\lambda_1(\Lambda_2)$ *obtained in Equation (12) and the contract parameters obtained in Equation (13) (resp. (14) and (15)) in the MNO's utility* U_1 *and derive it with respect to* Λ_2*. The traffic volumes that the MNO should allowed access to when he acts selfishly, are defined analytically for each contract as one of the positive roots of fourth order polynomial equations and solve* $\partial_{\Lambda_2}U_1\left(\Lambda_2^*(RS)\right) = 0$ *(resp.* $\partial_{\Lambda_2}U_1\left(\Lambda_2^*(QD)\right) = 0,\ \partial_{\Lambda_2}U_1\left(\Lambda_2^*(SR)\right) = 0$*).*

Proof of Proposition 2. The proof can be found in [11].

Table 1. Most profitable contract to use for the MNO depending on power relations and content/advertising investment level (RS means *revenue sharing*, QD means *quantity discount* and SR, *sales rebate*)

Advertising level \ MNO's power par.	$0 < \frac{1}{b} \leq 0.25$	$0.25 < \frac{1}{b} \leq 0.5$	$0.5 < \frac{1}{b} \leq 0.75$	$0.75 < \frac{1}{b} < 1$
$-0.1 \geq \theta > -0.5$	QD	SR	RS	RS
$-0.5 \geq \theta > -0.7$	QD	SR	SR	SR
$-0.7 \geq \theta \geq -1.0$	SR	SR	RS	RS

In Table 1 we identify the parameter $\frac{1}{b}$ with the MNO's power in the supply-chain; optimizing numerically both operators' consumers' retail prices, we determine which contract maximizes the MNO's share of the optimal supply-chain's profit depending on power relations and content/advertising investment level θ. The Table 1 results are logical since when the MNO is not powerful, he has to give the MVNO some incentives to collaborate, for example by using an access charge decreasing in the volume of traffic this latter sends on his network or by promising a refund if the MVNO's delay is above some predefined threshold; and when he is powerful, a revenue sharing contract is more lucrative since he gets a high fraction (equal to his power coefficient) of the supply-chain's total revenue. For example, in the last column of Table 1 (the MNO has high market power), we can explain the swapping between revenue sharing (RS) and sales rebate (SR) like this: when the investment is low, the MVNO is weak compared to the MNO so the MNO can impose risk sharing to the MVNO via a revenue sharing contract (RS); when the investment is higher, the MNO is threatened by the MVNO because it consumes more traffic (and so increases the average delay), so the MNO prefers a sales rebate contract (SR); finally when the investment is very high, the revenue of the MVNO is more attractive for the MNO than providing low delays in order to satisfy his consumers, so he would rather use a revenue sharing contract (RS).

4 Conclusions

In this paper, we have modeled the economic interactions between a MNO sharing his limited network resource and a MVNO lacking the infrastructure. We have designed an optimal pricing strategy when no traffic differentiation is introduced by the MNO while the MVNO invests in content/advertising to compensate for the QoS degradation. The access charge and the optimal maximal traffic volume that the MVNO should be allowed to send on the MNO's network to coordinate the supply-chain made of both operators and consumers, are defined for various popular contracts in the supply-chain literature (i.e. revenue sharing, quantity discount and sales rebate).

The model can be extended to include some dynamicity by introducing two competitive Markov decision processes modeling both operators' market share evolution, and uncertainty might result from the MNO's ignorance of the consumers' expected service level [6]. Besides the model could be transposed to the machine-to-machine market taking data integrity as QoS measure.

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References

[1] Acemoglu, D., Bimpikis, K., Ozdaglar, A.: Price and Capacity Competition. In: Games and Economic Behavior (2008) (to appear)
[2] Cachon, G.: Supply-chain Coordination with Contracts. Handbooks in Operations Research and Management Science 11, 227–339 (2007)
[3] Cachon, G., Lariviere, M.: Capacity Choice and Allocation: Strategic Behavior and Supply-Chain Performance. Management Science 45, 1091–1108 (1999)
[4] Sethi, S., He, X., Prasad, A.: Cooperative Advertising and Pricing in a Dynamic Stochastic Supply-Chain: Feedback Stackelberg Equilibrium. In: Production and Operations Management (2008) (to appear)
[5] Mutlu, H., Alanyali, M., Starobinski, D.: Spot Pricing of Secondary Spectrum Usage in Wireless Cellular Networks. In: IEEE INFOCOM (2008)
[6] Bernstein, F., Federgruen, A.: A General Equilibrium Model for Industries with Price and Service Competition. Operations Research 52, 868–886 (2003)
[7] Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E.: Designing and Managing the Supply-Chain: Concepts, Strategies and Case Studies. McGraw-Hill International Edition (2008)
[8] Myerson, R.: Game Theory: Analysis of Conflict. Harvard University Press, Cambridge (1991)
[9] Pohjola, O.-P., Kilkki, K.: Value-based methodology to analyze communication services. Netnomics 8, 135–151 (2007)
[10] Latouche, G., Ramaswami, V.: Introduction to Matrix Analytic Methods in Stochastic Modelling. In: Quasi-Birth-and-Death Processes, 1st edn., ch. 1. ASA SIAM, Philadelphia (1999)
[11] Le Cadre, H., Bouhtou, M., Tuffin, B.: A Pricing model for a Mobile Network Operator sharing limited resource with a Mobile Virtual Network Operator, Internal Report, Orange Labs/R&D/CORE/MCN/13.09